

# Day-ahead probabilistic load forecasting for individual electricity consumption – Assessment of point- and interval-based methods

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**Abstract**— The transition to high shares of renewables leads to high demand for flexibility in the energy system. Due to increasing use of heat pumps and electric cars, the residential sector becomes more and more relevant in this respect. A precondition for reliably providing flexibility is the detailed knowledge of the current and future electricity demand of the respective household, which becomes possible with the comprehensive rollout of smart metering systems. Since the load of individual households is highly volatile, probabilistic forecast methods—based on point prediction and direct interval prediction—are considered. The evaluations reveal that independently of the method, high interval widths result due to very volatile load patterns. All developed point prediction methods outperform the benchmark. The computation time can be reduced significantly by direct interval forecasting while maintaining comparable accuracy.

**Keywords**—smart metering systems, forecasting methods, residential sector, electrical load, demand side flexibility

## I. INTRODUCTION

Germany, together with 194 countries, has set the goal to keep “the increase in the global average temperature to well below 2°C above pre-industrial levels and pursuing efforts to limit the temperature increase to 1.5°C above pre-industrial levels, recognizing that this would significantly reduce the risks and impacts of climate change” [1]. This is an ambitious goal which implies an increasing extension of the installed net capacity of renewable (volatile) energies. To handle the discrepancy of demand and supply, flexibility becomes crucial. The future residential sector with heat pumps and electric cars can fulfil part of the necessary flexibility. To take advantage of the residential flexibility a detailed knowledge about the total electric load of this particular household is substantial (example: to prevent load shifting of the heat pump from time of small peaks to time of high peaks). With the data acquired by smart meters, detailed knowledge about the electricity demand of single households is available now. These data provide the possibility to determine the future potential flexibility. In this paper, day-ahead forecasting is analysed.

There is numerous literature about load forecasting. Most of the research, however, is done for aggregated load and not for individual households (for example [2], [3]). Furthermore, the focus of single-household load forecasting in literature is set on point forecasting [4], [5], [6]. Since the load of individual households is highly volatile and stochastic as a result of numerous influencing factors, these forecasting approaches entail the risk to greatly over- or underestimate the load which makes it problematic to use these predictions for example as electricity grid operator. In this paper, therefore, interval instead of pure point prediction is carried out. Two

approaches are compared: a probabilistic prediction approach based on point prediction and a direct interval prediction approach.

The paper is structured as follows. First, some methodological background about forecasting methods is provided and a review about existing work is presented. Furthermore, the developed methods for the probabilistic prediction based on point prediction and the direct interval prediction are explained (Section II.). The performance of both approaches is evaluated in a case study, and the results are compared (Section III.). The conclusion as well as future extensions are presented in Section IV.

## II. METHODOLOGY

There are two options of forecasting, point prediction and interval prediction. Point prediction forecasts one value for the load of every time step, while interval prediction yields an interval for the predicted load with a given probability. A typical application for interval predictions are weather forecasts. Hereby, they determine for example whether it will rain the next day or not with a certain probability. Since the load of individual households is highly volatile and stochastic, probabilistic instead of point forecast methods are considered in this paper. Two approaches of interval prediction are evaluated: probabilistic forecast based on point prediction and direct interval prediction.

### A. Probabilistic prediction based on point prediction

There are several methods for point prediction. Examples are point predictions based on artificial neural networks or regression methods. An overview of different point prediction methods is given in [7]. According to [7], there are three different categories for point prediction. The first ones are reference-based methods, in particular methods based on typical load profiles or comparative-day methods. These are the simplest methods and already widely used for aggregated loads [7]. The second category are classical methods, which include time series analysis, regression methods and Kalman filters. Classical methods use mathematical models which predict the load based on historical data and at least one external parameter. The parameters of the models are determined with a training set and for example the least square method. In [5], a linear regression model is applied for load forecasting. The last category is artificial intelligence with the most popular method based on artificial neural networks. But also, support vector machines (SVM) [8] and expert-systems [9] belong in this category [7]. For artificial intelligence, no analytical model is built, therefore, it is the preferable method when no knowledge exists about the detailed relation. Typical artificial neural networks for load prediction are neural network ensembles (with bagging) (see [10] and [11]) and (multi-layer) feed-forward neural networks (see [12] and [13]).

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In this paper, the point forecast methods are reference-based and use data from recent days (previous day and previous week) and load profiles generated from historical data of this particular household for a type of day (standard load profile - SLP) as input data to forecast the load of the next day. For real-life application of the methods, only data until 12 pm on the previous day are available, for the remaining hours data from yesterday are taken. Reason for that is the deadline of the day-ahead market. In total, six point forecast methods are applied which combine the two methods of the reference based techniques. A naive forecast considering the load of the last day is applied as a benchmark for the point forecast methods. An overview over the selected point prediction methods is given in Table I. Hereby,  $E$  is load,  $P$  prediction method, SLP typical load profile of a particular household for the type of day. The input data are weight by weighting parameters  $a_i, b_i, c_i, d_i$  which are determined in a separate optimisation using training data.  $mean(E_X)$  or  $max(E_X)$  is in terms of the mean/ max load of this particular day. The point prediction is carried out for every time step.

TABLE I. POINT FORECAST METHODS

	Equation	Weighting parameters
$P_1$	$P_1 = a_1 E_{SLP} + a_2 E_{previous-day} + a_3 E_{previous-week}$ $a_1 + a_2 + a_3 = 1$	$a_1, a_2, a_3 \in [0.1, 0.99]$
$P_5$	$P_5 = E_{SLP} \frac{b_1 mean(E_{previous-day}) + b_2 mean(E_{previous-week})}{mean(E_{SLP})}$ $b_1 + b_2 = 1$	$b_1, b_2 \in [0.1, 0.99]$
$P_6$	$P_6 = c_1 max(E_{SLP}, E_{previous-day}, E_{previous-week}) + c_2 E_{SLP}$ $c_1 + c_2 = 1$	$c_1, c_2 \in [0.01, 0.99]$
$P_7$	$P_7 = \begin{cases} (1 + d_1) E_{SLP} \frac{max(E_{previous-day})}{mean(E_{previous-day})} & E_{previous-day} > 2 E_{SLP} \\ (1 + d_2) E_{SLP} \frac{max(E_{previous-week})}{mean(E_{previous-week})} & E_{previous-week} > 2 E_{SLP} \\ E_{SLP} & else \end{cases}$	$d_1, d_2 \in [0.01, 1]$
$P_{naive}$	$P_{naive} = E_{previous-day}$	-
$P_{SLP}$	$P_{SLP} = E_{SLP}$	-

With these point predictions, probabilistic prediction shall be developed. The method for the generation of a probabilistic prediction based on point prediction is obtained from [14] and [15]. El-Baz et al. developed a probabilistic PV generation forecast for energy management systems based on the distribution function on the prediction error of a training set. Firstly, a point prediction based on a clear sky model was developed. Since clouds have a major effect on PV performance, the forecasted cloudiness is taken into account. Eight categories of cloudiness were defined. For every category, a distributed function of the difference between the forecasted PV generation and the real PV measurements (prediction error) was determined training an ensemble of

regression trees. With these distribution functions, the point prediction is scaled. This way, for a fixed probability a prediction can be made, for several fix probabilities a probabilistic field is obtained (cf. Fig.1).

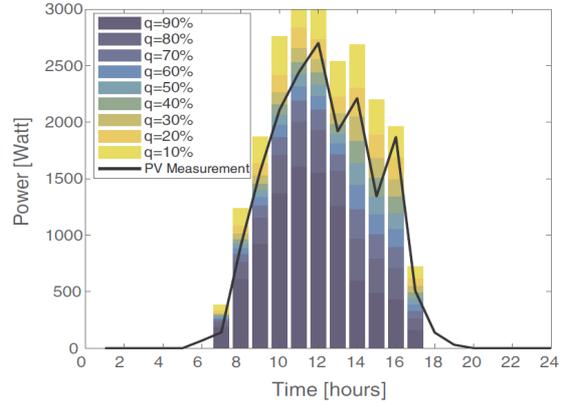


Fig.1. Probabilistic PV generation acc. to [12].

To adjust this concept for load forecasting, not the cloudiness but the type of the season/day/daytime is taken as an input. There are three seasons (winter, summer and transition), three days (Saturday, Sunday and weekday) and two daytimes (day and night). These add up to in total 18 categories. Furthermore, an interval and not several probabilistic bands are needed, so the probabilities 5 % and 95 % are taken to generate an interval of 90 %. Moreover, this method is applied for several point prediction methods (see Table III). The flow chart of the developed method is shown in Fig.2.

### B. Direct interval prediction

In the first approach, the probabilistic prediction is carried out by first generating a point prediction, which provides an interval prediction using the distribution functions of a training set. A straighter approach is to generate directly an interval prediction without the point prediction as an intermediate stage. One possible way is the following, which utilizes only recent data (see Fig.3): For the interval prediction for one particular time step (for example 6:00), recent data from the same time step as well as adjacent ones are taken. One option (in the following case 1) are the time steps 5:45, 6:00 and 6:15 of the previous day as well as the previous week and the week before ('non-scaled interval prediction'). Of

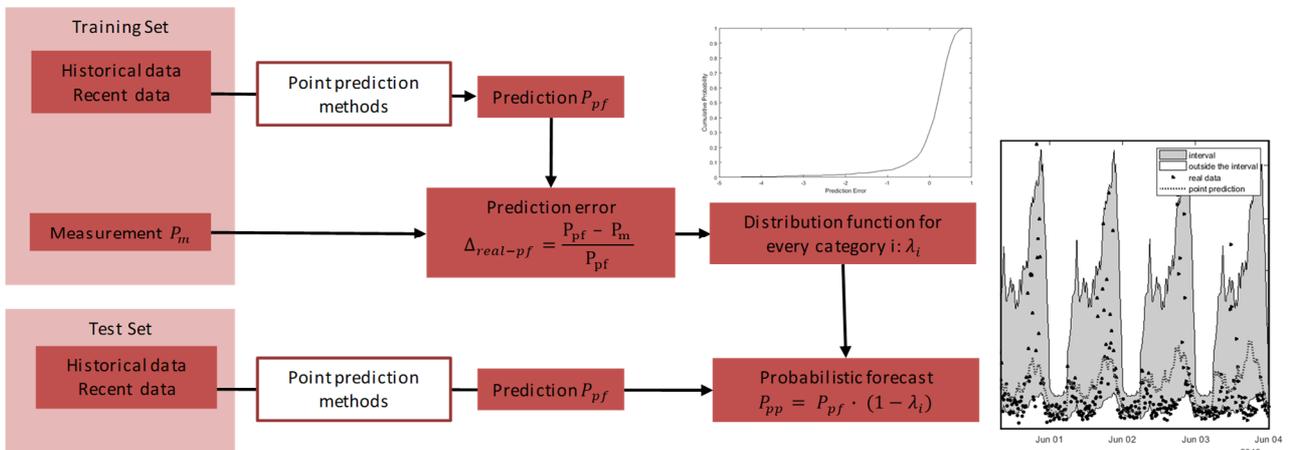


Fig. 2. Flow chart describing the process of the probabilistic load forecast based on point prediction

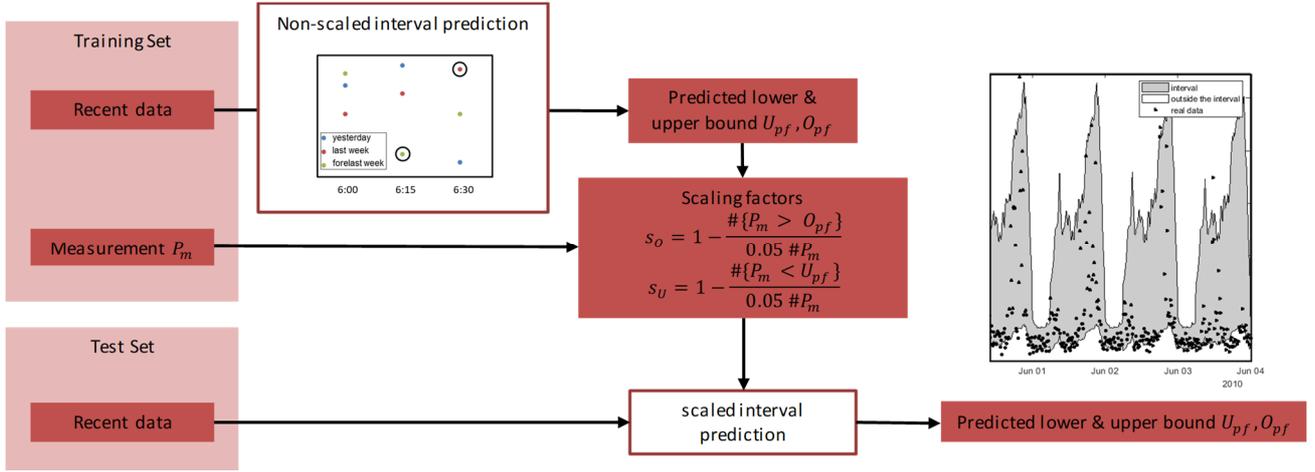


Fig.3. Flow chart describing the process of the direct interval load forecast

these nine data points, the second largest as well as the second smallest are chosen as upper and lower bound ('predicted lower & upper bound  $U_{pf}, O_{pf}$ '). This is done for the training set providing a fixed but unknown interval. As a certain interval is desired (for example 90 %), the lower and upper bound are scaled with  $s_o$  and  $s_u$  ('scaling factors'). The same scaling factors  $s_o$  and  $s_u$  are then used for the test data. Alternatively to case 1, other data point selection methods are possible. In the following case study, the six cases shown in Table II are applied. The first row indicates which time steps, the second row which days are to take. The fourth row indicates which data points are selected as upper and lower bound. And last, the third row shows how many data points are in total and how many are in between the upper and lower bound (in brackets).

### III. RESULTS

To evaluate the different forecasting methods, a case study is carried out consisting of 74 households in Germany with a resolution of 15 minutes for one year (2010). The mean load of the different households varies between 160 W and almost 1 kW and differs notably between the different households. The yearly electricity demand differs between 1400 kWh/a and 8635 kWh/a with a mean value of 4685 kWh/a. The data seems regularly distributed (see Fig.4). The mean value corresponds mostly to the yearly electricity demand of an accommodation unit which lies around 3350 kWh/a (calculation based on [16]). The value is slightly higher since only single family houses are included in the data which have a higher energy consumption than multi-family houses (which are also considered in the calculation based on [16]). Due to the variation of load between the different households, normalized deviations are used for evaluation. The data are split in a training set and a test set. There are training and test data for every season; approx. 300 days are taken as training data and approx. 50 days are used as test data. The first two

TABLE II. DIFFERENT CASES OF DATA SELECTION FOR THE DIRECT INTERVAL PREDICTION

Case	Points per day	Day selection	# data points	lower & upper bound
1	i-1, i, i+1	d-1, d-7, d-14	9 (7)	second smallest / second largest
2	i-2, i-1, i, i+1, i+2	d-1, d-7	10 (8)	second smallest / second largest
3	i-1, i, i+1	d-1, d-2, d-7	9 (7)	second smallest / second largest
4a	i-2, i-1, i, i+1, i+2	d-1, d-2, d-7, d-14	20 (16)	third smallest / third largest
4b	i-2, i-1, i, i+1, i+2	d-1, d-2, d-7, d-14	20 (14)	fourth smallest / fourth largest
4c	i-2, i-1, i, i+1, i+2	d-1, d-2, d-7, d-14	20 (12)	fifth smallest / fifth largest

weeks are not considered since there are not enough historical data available.

For the comparison of the predicted intervals, the interval widths ( $I_{max}, I_{mean}, I_{median}$ ) and the degree of fulfilment ( $f_{mean}, \Delta f$ ) are analysed. To check the different point predictions against the benchmark, mean absolute percentage error (MAPE), mean absolute error (MAE), root mean square error (RMSE) and the maximal absolute error ( $aE_{max}$ ) are utilized. Furthermore, the calculation time ( $t_c$ ) is considered as an additional performance parameter.

TABLE III. RESULTS - ERROR EVALUATION OF THE INTERVAL PREDICTION

	Point-based prediction methods					Direct interval methods					
	$P_{SLP}$	$P_5$	$P_1$	$P_6$	$P_7$	case 1	case 2	case 3	case 4a	case 4b	case 4c
$I_{median}$	1.99	1.76	1.95	1.97	1.97	1.92	1.85	1.92	1.86	1.85	1.87
$I_{mean}$	2.14	2.00	2.11	2.12	2.13	3.02	2.76	2.94	2.72	2.72	2.61
$I_{max}$	8.31	9.63	8.7	8.45	8.62	24.63	18.49	23.24	17.41	17.8	17.57
$f_{mean}$	90.1 %	90.1 %	90.1 %	90.1%	90.1 %	89.7 %	89.7 %	89.7 %	89.8 %	89.7 %	89.7 %
$\Delta f$	16.0 %	9.9 %	14.8 %	15.5 %	15.8 %	6.0 %	6.4 %	7.2 %	7.4 %	6.9 %	6.9 %

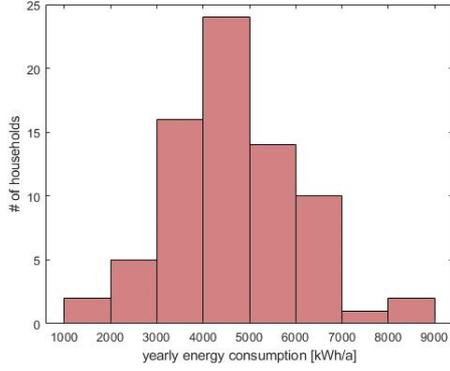


Fig. 4. Electricity demand of the 74 households

The evaluations (see Table III) reveal that the degree of fulfilment is reached within a tolerance for all prediction methods ( $f_{mean}$  is around 90% for all methods).

Independently of the method, high interval widths are calculated due to very volatile load patterns. For  $I_{max}$ , however, higher values are obtained for the direct interval prediction method. This is caused by outliers in the considered recent days which lead to improbably high predicted upper bounds, since no correction with mean values is applied. To improve the results, the maximum upper bound can be restricted to the maximum historical load scaled with a factor to prevent this behaviour. A sensitivity study shows that the rate of fulfilment is still reached and  $I_{max}$  decreases notably by using this improved method.

The direct interval prediction methods, however, outperform the point-based probabilistic methods on the variation of the degree of fulfilment  $\Delta f$ . It varies between 6 % and 7.5 %, while for the point-based prediction methods it ranges between 8.5 % and almost 16 %, where the high variations are caused only by very few households. This shows that the point-based methods are more sensitive to certain unusual types of load patterns. Since the point prediction methods are based on typical load profiles, this is an indication that load profiles, which do not adhere to a typical load profile, are not well predictable with these methods. Fig.5 shows two load profiles which are not suitable for the point-based prediction methods and zoom in on the atypical interval of the load. For household 68, ‘summer holidays at home’ with very high electrical load can be seen. Household 9 shows ‘summer holidays away’ with an atypical

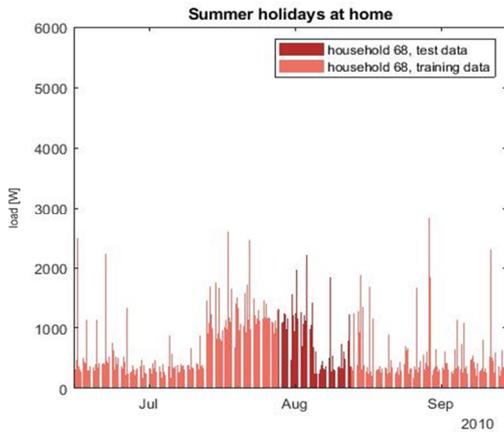


TABLE IV. RESULTS - ERROR EVALUATION OF THE POINT BASED PREDICTION

	$P_{naive}$	$P_{SLP}$	$P_5$	$P_1$	$P_6$	$P_7$
$aE_{max}$	10.59	9.01	8.96	9.07	899	8.98
$SMAPE$	53 %	47 %	45 %	46 %	47 %	47 %
$RMSE$	1.26	0.87	0.85	0.86	0.87	0.87
$MAE$	0.66	0.50	0.48	0.50	0.51	0.50

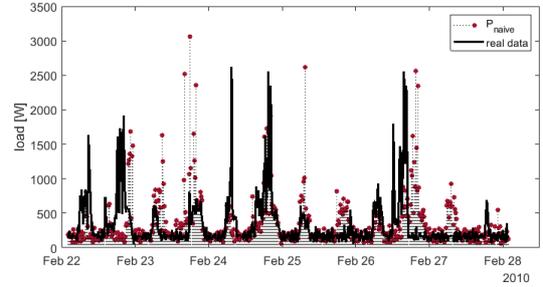


Fig. 6. Exemplary days for the benchmark (naive prediction)

decrease of load for half a month. For household 68, low degree of fulfilment is reached due to the decrease of load within the test data. The interval does not follow that change fast enough, which results in a very high lower bound. The same holds for household 9: due to low load during the summer holidays, the lower bound is too high as well.

The last parameter to evaluate is the calculation time  $t_c$ . It can be reduced significantly by directly forecasting intervals, taking less than 2 min in comparison to hours of computation time for the point-based methods. The reason for that is mostly the weighting parameter optimisation (not existing for  $P_{naive}$  and  $P_{SLP}$ ) and to a smaller extent the algorithm itself. Since point prediction has to be carried out first, then distribution functions have to be calculated,  $t_c$  increases. To save computation time, the weighting parameter optimisation might be replaced by empirical values, which will—depending on the empirical method—decrease the forecasting performance (slightly), but significantly decreases  $t_c$ .

Now, a detailed analysis is performed for the different point prediction methods (see Table IV). All developed point prediction methods outperform the benchmark. The detailed analysis of the results reveals that the base load can be predicted very well while the load peaks of real data and forecasted data are seldom matched (see exemplary a few days predicted by  $P_{naive}$ , Fig.6). The optimisation of the weighting

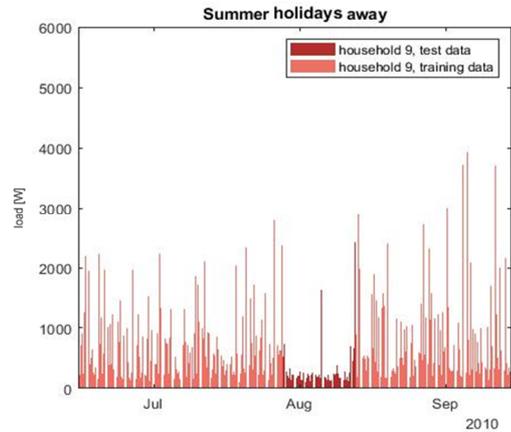


Fig. 5. Households with atypical pattern

parameters of the more advanced methods therefore always weight the parts for the base load forecasting and set the other weighting parameter in most cases only slightly larger than the minimum value. The naive prediction which tries to anticipate the peaks and the base load has no option to weight and therefore reaches higher errors than the more advanced methods. The best point prediction is a scaled (by recent data) load profile generated from historical data ( $P_5$ ). This method outperforms a simple load profile generated from historical data without considering recent influences. All developed point prediction methods are quite close to each other.

Considering the results of Table III and Table IV,  $P_5$  is the preferred point prediction method with good performance in both point and interval prediction. Regarding the direct interval forecasting, the best results are obtained with *case 2* and *4* (there are only small differences between the variants *4a*, *4b* and *4c*). It shows that the direct interval prediction improves with more available data points. The selection whether to take  $P_5$ -based probabilistic prediction or the direct interval prediction with *case 2* or *4* depends on the use case. If computing power and storage capacity are limited (small electronic device without Wi-Fi), the direct interval prediction is to choose since only a small amount of data is necessary and the calculation is comparably fast. Furthermore, it is to be preferred when the data show less tendency to typical load profiles. If the application requires both point and interval predictions (e.g. for visualization), the point-based probabilistic prediction with  $P_5$  is recommended.

#### IV. CONCLUSION

With the transition to high shares of renewables, the need for flexibility increases. The residential sector offers a high potential for demand-side flexibility due to increasing use of heat pumps and electric cars. For providing reliable flexibility, load forecasting of the total energy demand of the particular household is crucial. To take the high volatility of electrical load for single households into account, probabilistic instead of point forecast methods are considered. In this paper, different day-ahead probabilistic load forecasting methods for individual electricity consumption are developed. There are based on two different approaches; the first approach develops a point forecast which is subsequently transformed to an interval prediction. The second approach is a direct interval prediction. These two approaches are compared and analysed with a case study of 74 households in Germany for one year.

The evaluations reveal that independently of the method, high interval widths result due to very volatile load patterns. Regarding the grade of fulfilment, the direct interval prediction methods show a more stable result. The variation of the grade of fulfilment between the different households differs around 6 % in comparison to more than 10 % for most point-based methods. In addition, the computation time for point-based methods is very high, while the direct interval prediction is quick and requires less data. An advantage of the point-based method is the point prediction itself, if this is desired as additional output. All developed point prediction methods outperform the benchmark for all error evaluation.

There is room for improvement for both methods. To reduce the maximum interval width, a limit based on the maximum of the training set of this particular household and the maximum of the used recent data can be developed. This

would improve the direct interval prediction methods. To improve the variation of the grade of fulfilment, a pre-sorting of the households with deleting all households with no typical load pattern can be done.

As a next step, a comparison to alternative forecast methods based on artificial neural networks is intended. Furthermore, a detailed analysis about the performance of every single household is planned. This contributes to the research question whether specific load patterns and characteristics indicate forecasting performance.

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